Analysis and Optimization of Asymmetric Epicyclic Gears

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Following the Direct Gear Design approach to asymmetric epicyclic gear stages with singular and compound planet gears, methods of optimization of the tooth flank asymmetry factor and root fillet profile are considered.

THE DESIGN OBJECTIVE OF ASYMMETRIC TOOTH gears is to improve performance of the primary drive flank profiles at the expense of the opposite coast profiles' performance. The coast flanks are unloaded or lightly loaded during a relatively short work period. Such gears should be used for unidirectional or mostly unidirectional load transmission when the coast flanks are unloaded or lightly loaded during relatively short work time.

In a simple epicycling gear stage, the singular planet gear transmits the same load by both tooth flanks. However, the convex planet gear tooth flanks are in simultaneous meshes with the convex sun gear tooth flanks and the concave ring gear tooth flanks resulting in very different contact stress values in those meshes.

In epicycling gear drives with compound planet gears, one portion of the planet gear is in mesh with the sun gear and other one with the ring gear. In this case, the coast tooth flanks are used only when the gear drive works in a reversed load direction.

This paper describes a gear geometry analysis and the asymmetry factor and tooth root fillet optimization of epicycling stages with singular and compound planet gears. This allows for a considerable increase in power transmission density, increase in load capacity, and reduction in the size and weight of asymmetric epicyclic gear drives.

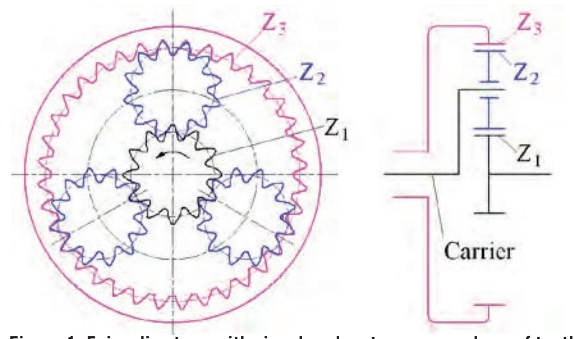


Figure 1: Epicyclic stage with singular planet gears; numbers of teeth: z_1 — sun gear, z_2 — planet gear, and z_3 — ring gear

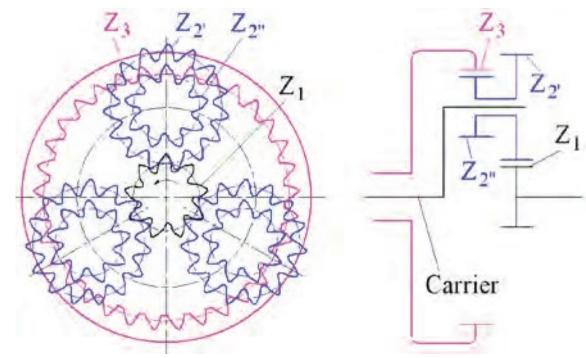


Figure 2: Epicyclic stage with compound planet gears; numbers of teeth: z_1, — sun gear, $z_{2'}$ — planet gear portion engaged with sun gear, $z_{2''}$ — planet gear portion engaged with ring gear, and z_3 — ring gear

BASIC ASYMMETRIC EPICYCLIC GEAR STAGES

Epicyclic gear stages provide high load capacity and compactness to gear drives. There is a huge variety of different combinations of planetary gear arrangements. Although some of them are quite complicated, they typically contain simple epicyclic stages with singular planet gears (see Figure 1) or epicyclic stages with compound planet gears (see Figure 2). For an epicyclic planetary stage with singular planet gears where the ring gear is stationary, the practical gear ratio varies from 3:1 to 9:1; for an epicyclic planetary stage with compound planet gears, the practical gear ratio varies from 8:1 to 30:1 [1].

The gear ratio of an epicyclic gear stage depends on its arrangement. The most common arrangements are the solar stage with the stationary carrier, the planetary stage with the stationary ring gear, and the differential stage where the carrier and ring gear are rotating with the same rpm but in opposite directions. Table 1 presents the gear ratio equations for different epicyclic arrangements.

ASYMMETRY FACTOR OPTIMIZATION

Epicyclic Stage with Singular Planet Gears

In an epicyclic stage with singular planet gears (see Figure 3), opposite flanks of the planet gear are in simultaneous contact with the sun and ring gears. In this case, the goal

Stage Arrangement	with singular planet gears	with compound planet gears
Solar	$u=-z_3/z_1$	$u=-z_3 z_{2'}/(z_{2''}z_1)$
Planetary	$u=1+z_3/z_1$	$u=1+z_3 z_{2'}/(z_{2''}z_1)$
Differential	$u=\pm(1+2z_3/z_1)$	$u=\pm(1+2z_3 z_{2'}/(z_{2''}z_1))$

Table 1

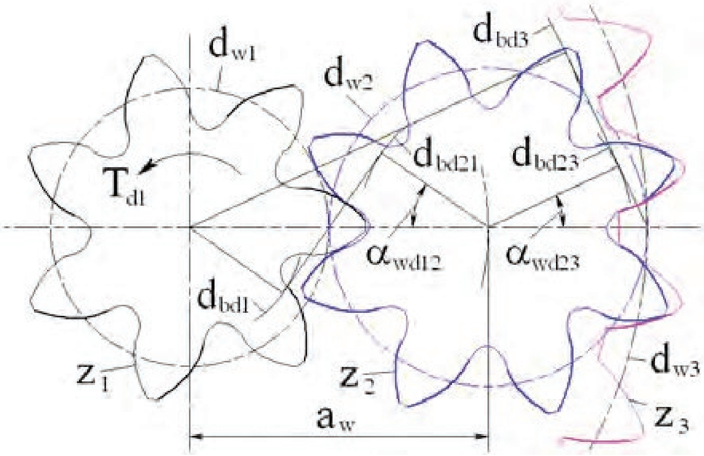


Figure 3: Epicyclic mesh with singular planet gears

of asymmetry factor optimization is equalization of the contact stress safety factors in the external mesh of the sun and planet gears and the internal mesh of the planet and ring gears.

The flank contact stress safety factor is:

$$S_H = \frac{\sigma_{HP12}}{\sigma_{H12}} = \frac{\sigma_{HP23}}{\sigma_{H23}} \quad \text{Equation 1}$$

where σ_{HP12} and σ_{HP23} are permissible contact stress in the sun/planet gear mesh and the planet/ring gear mesh, and σ_{H12} and σ_{H23} are maximum operating contact stress in the sun/planet gear mesh and the planet/ring gear mesh.

Equation 1 can be converted into the operating contact stress ratio:

$$\frac{\sigma_{H23}}{\sigma_{H12}} = \frac{\sigma_{HP23}}{\sigma_{HP12}} \quad \text{Equation 2}$$

In order to simplify the asymmetry factor optimization, the maximum contact stresses in the sun/planet and planet/ring gear meshes are replaced with the contact stress at the pitch points. The operating contact stresses at the pitch points (in the sun/planet gear mesh) are:

$$\sigma_{H12} = z_E \frac{2}{d_{w1}} \sqrt{\frac{2T_{d1}}{b_{w12}\sin(2\alpha_{wd12})} \frac{u_{12}+1}{u_{12}}} \quad \text{Equation 3}$$

where Z_E — elastic factor (material property factors), $u_{12} = z_2/z_1$ — sun/planet mesh gear ratio, b_{w12} — contact face width, α_{wd12} — drive flank pressure angle, T_{d1} — sun gear driving torque (in the sun/planet gear mesh):

$$\sigma_{H23} = z_E \frac{2}{u_{12}d_{w1}} \sqrt{\frac{2u_{12}T_{d1}}{b_{w23}\sin(2\alpha_{wd23})} \frac{u_{23}-1}{u_{23}}} \quad \text{Equation 4}$$

where $u_{23} = z_3/z_2$ — planet/ring mesh gear ratio, b_{w23} — contact face width, α_{wd23} — drive flank pressure angle.

Assuming all gears are made from the same material and inserting Equations 3 and 4 into Equation 2, the coefficient A is equal to:

$$A = \frac{\sin(2\alpha_{wd23})}{\sin(2\alpha_{wd12})} = \frac{b_{w12}}{b_{w23}} \frac{u_{23}-1}{u_{23}(u_{12}+1)} \frac{\sigma^2_{HP12}}{\sigma^2_{HP23}} \quad \text{Equation 5}$$

According to Reference [2], "The permissible stress at limited service life or the safety factor in the limited life stress range is determined using life factor Z_{NT}." This allows replacing the permissible contact stresses in Equation 18 with life factors:

$$A = \frac{\sin(2\alpha_{wd23})}{\sin(2\alpha_{wd12})} = \frac{b_{w12}}{b_{w23}} \frac{u_{23}-1}{u_{23}(u_{12}+1)} \frac{Z^2_{NT12}}{Z^2_{NT23}} \quad \text{Equation 6}$$

where z_{NT12} and z_{NT23} are the life factors in the sun/planet and planet/ring gear meshes respectively. These life factors can be defined by numbers of load cycles. The load cycle number ratio in an epicyclic stage with singular planet gears is:

$$\frac{N_{12}}{N_{23}} = \frac{z_3}{z_1} \quad \text{Equation 7}$$

where N_{12} and N_{23} are the numbers of load cycles in the sun/planet and planet/ring gear meshes respectively. Then, the asymmetry factor [3] is:

$$K = \frac{\cos\alpha_{wd23}}{\cos\alpha_{wd12}} = \frac{\sqrt{1+\sqrt{1-A^2(\sin 2\alpha_{wd12})^2}}}{\sqrt{2}\cos\alpha_{wd12}} \quad \text{Equation 8}$$

For example: The sun gear number of teeth is $n_1 = 37$, the planet gear number of teeth is $n_2 = 32$, the output gear number of teeth is $n_3 = 101$, the contact face width ratio is $b_{w12}/b_{w23} = 1.4$, the number of load cycles in the sun/planet gear mesh is $n_{12} = 10^9$, the number of load cycles in the planet/ring gear mesh is $n_{23} = n_{12} \times z_1/z_3 = 3.7 \times 10^8$. This makes the life factors for nitrocarburized steel $Z_{NT12} = 0.89$ and $Z_{NT23} = 0.90$, and the factor A = 0.672. If the selected sun/planet gear mesh drive pressure angle $\alpha_{wd12} = 36°$, the asymmetry coefficient is K = 1.16 and the planet/ring gear mesh drive pressure angle is $\alpha_{wd23} = 19.8°$.

Epicyclic Stage with Compound Planet Gears

In an epicyclic stage with compound planet gears (see Figure 2), one portion of the planet gear is in mesh with the sun gear and the other with the ring gear. In this case, the coast tooth flanks are used only when the gear drive works in a reversed load direction. The goal of asymmetry factor optimization is equalization of the contact stress safety factors in the main drive and opposite reversed load transmission directions.

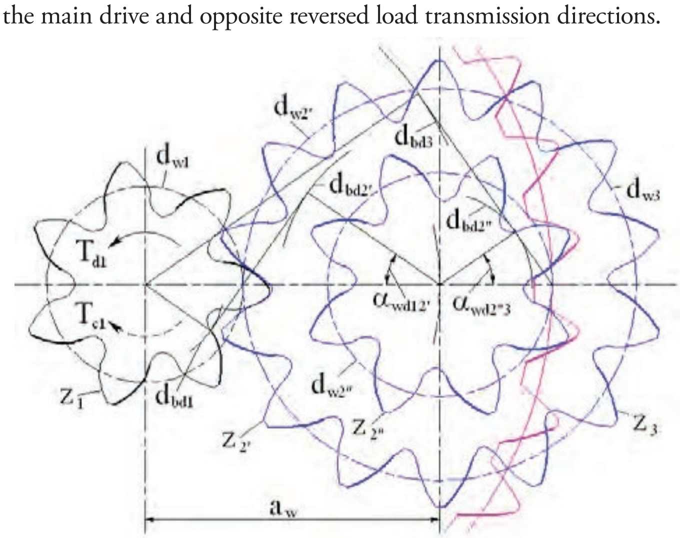


Figure 4: Epicyclic mesh with compound planet gears

Then, the flank contact stress safety factor is:

$$S_H = \frac{\sigma_{HPd}}{\sigma_{Hd}} = \frac{\sigma_{HPc}}{\sigma_{Hc}} \quad \textbf{Equation 9}$$

where σ_{HPd} and σ_{HPc} are permissible contact stress in the main drive and opposite reversed load transmission directions, and σ_{Hd} and σ_{Hc} are maximum operating contact stress in the main drive and opposite reversed load transmission directions.

Equation 9 can be converted into the operating contact stress ratio:

$$\frac{\sigma_{Hd}}{\sigma_{Hc}} = \frac{\sigma_{HPd}}{\sigma_{HPc}} \quad \textbf{Equation 10}$$

In order to simplify the asymmetry factor optimization, maximum contact stresses are replaced with the contact stress at the pitch points. The operating contact stresses at the pitch points (in the main drive load transmission direction) are:

$$\sigma_{Hd12'} = z_E \frac{2}{d_{w1}} \sqrt{\frac{2T_{d1}}{b_{w12'} \sin(2\alpha_{wd12'})} \frac{u_{12'}+1}{u_{12'}}} \quad \textbf{Equation 11}$$

$$\sigma_{Hd2"3} = z_E \frac{2}{d_{w2"}} \sqrt{\frac{2u_{12'}T_{d1}}{b_{w2"3} \sin(2\alpha_{wd2"3})} \frac{u_{2"3}-1}{u_{2"3}}} \quad \textbf{Equation 12}$$

where $u_{12'} = z_{2'}/z_1$ — sun/planet mesh gear ratio, $u_{2"3} = z_3/z_{2"}$ — planet/ring mesh gear ratio, $z_{2'}$ — tooth numbers of the planet gear engaged with the sun gear, $z_{2"}$ — tooth numbers of the planet gear engaged with the ring gear, $b_{w12'}$ and $b_{w2"3}$ — contact face widths in the sun/planet gear mesh and the planet/ring gear mesh, $\alpha_{wd12'}$ and $\alpha_{wd2"3}$ — drive flank pressure angles in the sun/planet and the planet/ring gear meshes respectively, and T_{d1} — sun gear driving torque (in the main drive load transmission direction):

$$\sigma_{Hc12'} = z_E \frac{2}{d_{w1}} \sqrt{\frac{2T_{c1}}{b_{w12'} \sin(2\alpha_{wc12'})} \frac{u_{12'}+1}{u_{12'}}} \quad \textbf{Equation 13}$$

$$\sigma_{Hc2"3} = z_E \frac{2}{d_{w2"}} \sqrt{\frac{2u_{12'}T_{c1}}{b_{w2"3} \sin(2\alpha_{wc2"3})} \frac{u_{2"3}-1}{u_{2"3}}} \quad \textbf{Equation 14}$$

where $\alpha_{wc12'}$ and $\alpha_{wc2"3}$ — coast flank pressure angles in the sun/planet and the planet/ring gear meshes respectively and T_{c1} — sun gear reversed torque.

Assuming all gears are made from the same material and inserting Equations 11-14 into Equation 10, the coefficient A is equal to:

$$A = \frac{\sin(2\alpha_{wc12'})}{\sin(2\alpha_{wd12'})} = \frac{\sin(2\alpha_{wc2"3})}{\sin(2\alpha_{wd2"3})} = \frac{T_{c1}}{T_{d1}} \frac{\sigma^2_{HPd}}{\sigma^2_{HPc}} \quad \textbf{Equation 15}$$

The permissible contact stresses in Equation 15 can be replaced for life factors:

$$A = \frac{\sin(2\alpha_{wc12'})}{\sin(2\alpha_{wd12'})} = \frac{\sin(2\alpha_{wc2"3})}{\sin(2\alpha_{wd2"3})} = \frac{T_{c1}}{T_{d1}} \frac{Z^2_{NTd}}{Z^2_{NTc}} \quad \textbf{Equation 16}$$

where Z_{NTd} and Z_{NTc} are the life factors in the main drive and opposite reversed load transmission directions respectively, which are defined by N_d and N_c — the numbers of load cycles in the main drive and opposite reversed load transmission directions.

Then, the asymmetry factors (in the sun/planet gear mesh) are:

$$K_{12'} = \frac{\cos\alpha_{wc12'}}{\cos\alpha_{wd12'}} = \frac{\sqrt{1+\sqrt{1-A^2(\sin 2\alpha_{wd12'})^2}}}{\sqrt{2}\cos\alpha_{wd12'}} \quad \textbf{Equation 17}$$

and (in the planet/ring gear mesh):

$$K_{2"3} = \frac{\cos\alpha_{wc2"3}}{\cos\alpha_{wd2"3}} = \frac{\sqrt{1+\sqrt{1-A^2(\sin 2\alpha_{wd2"3})^2}}}{\sqrt{2}\cos\alpha_{wd2"3}} \quad \textbf{Equation 18}$$

Unlike in the epicyclic asymmetric gear stage with singular planet gears, the optimized asymmetry coefficients depend only on the main driving and reversed torque ratio and numbers of cycles in the main drive and reversed load transmission directions.

For example: Main driving and reversed torque ratio is T_{d1}/T_{c1} = 1.5, the number of load cycles in the main drive load transmission direction is $N_d = 10^9$, the number of load cycles in the reversed load transmission direction is $N_c = 10^7$. This makes the life factors for nitrocarburized steel Z_{NTd} = 0.89 and Z_{NT23} = 0.97, and the factor A = 0.561. If the selected sun/planet gear mesh drive pressure angle $\alpha_{wd12'}$ = 36°, its asymmetry coefficient is $K_{12'}$ = 1.187 and the coast pressure angle in this mesh is $\alpha_{wc12'}$ = 16.1°. If the selected planet/ring gear mesh drive pressure angle $\alpha_{wd2"3}$ = 33°, its asymmetry coefficient

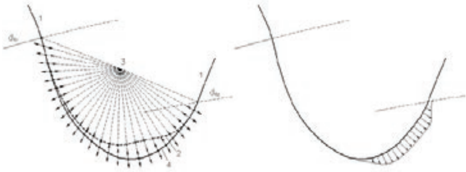


Figure 5: Tooth root fillet optimization: a — profile shaping technique, b — stress chart; 1 — involute tooth flanks, 2 — initial fillet profile, 3 — fillet center, 4 — optimized fillet profile, d_{fd}, d_{fc} — form circle diameters of the drive and coast tooth flanks

is $K_{2''3}$ = 1.147 and the coast pressure angle in this mesh is $\alpha_{wc2''3}$ = 15.4°.

TOOTH ROOT FILLET OPTIMIZATION

In Direct Gear Design, the tooth fillet is designed after the involute flank parameters are completely defined. The goal is to achieve minimal stress concentration in the tooth fillet profile. In other words, the maximum bending stress should be evenly distributed along a large portion of the fillet. The initial fillet profile is a trajectory of the mating gear tooth tip in a tight (zero backlash) mesh. This allows for an exclusion of interference with the mating gear tooth.

The fillet optimization method that is used in Direct Gear Design was developed by Dr. Yuriy V. Shekhtman [4]. This method utilizes the following calculation processes:

- Definition of a set of mathematical functions that is used to describe the optimized fillet profile. Parameters of these functions are defined during the optimization process.
- Stress is calculated using a 2D finite element analysis (FEA) subroutine that achieves satisfactory optimization results within a reasonable amount of time.
- A random search method is used to define the next step in the multi-parametric iteration process of fillet profile optimization.

The fillet optimization method establishes the approximate fillet center (see Figure 5). The first and last finite element nodes of the initial fillet profile located on the form diameter circle cannot be moved during the optimization process. The rest of the initial fillet finite element nodes are moved along straight lines that pass through the fillet center. Bending stresses are calculated for every iteration of the fillet profile configuration. Variable parameters of the fillet profile functions that describe the fillet profile for the next iteration are defined depending on stress calculation results of the previous iteration. If it provided a stress reduction, the optimization process moves fillet nodes in the same direction. If stress was increased, the nodes are moved in the

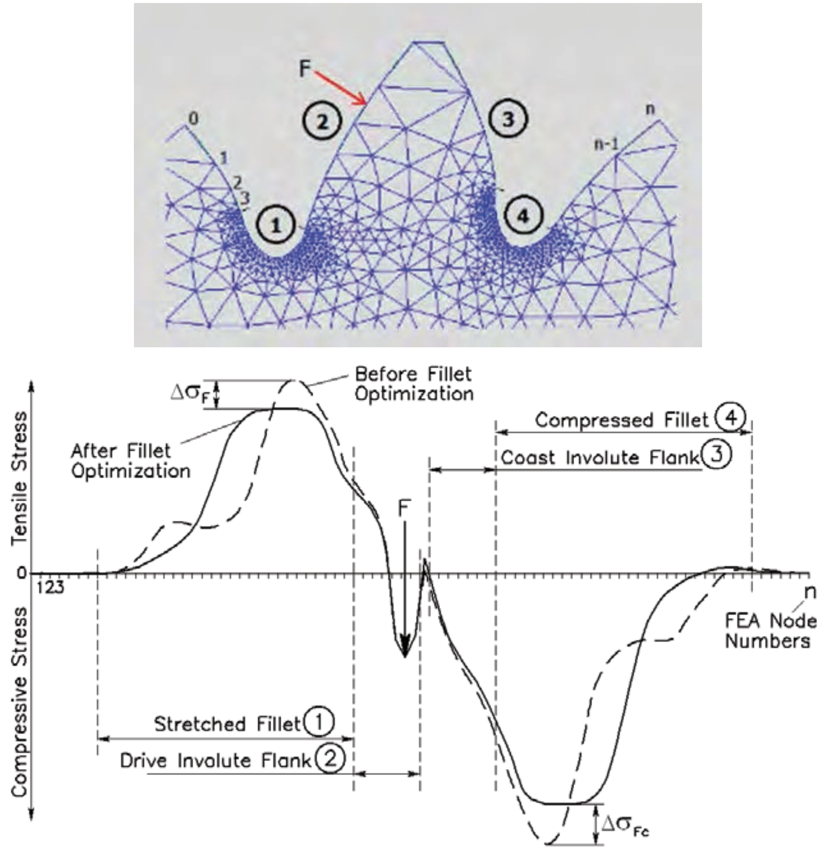


Figure 6: Tooth stress distribution comparison before and after root fillet optimization: a — tooth FEA mesh, b — stress distribution charts, $\Delta\sigma_F$ — tensile stress reduction, $\Delta\sigma_{FC}$ — compressive stress reduction

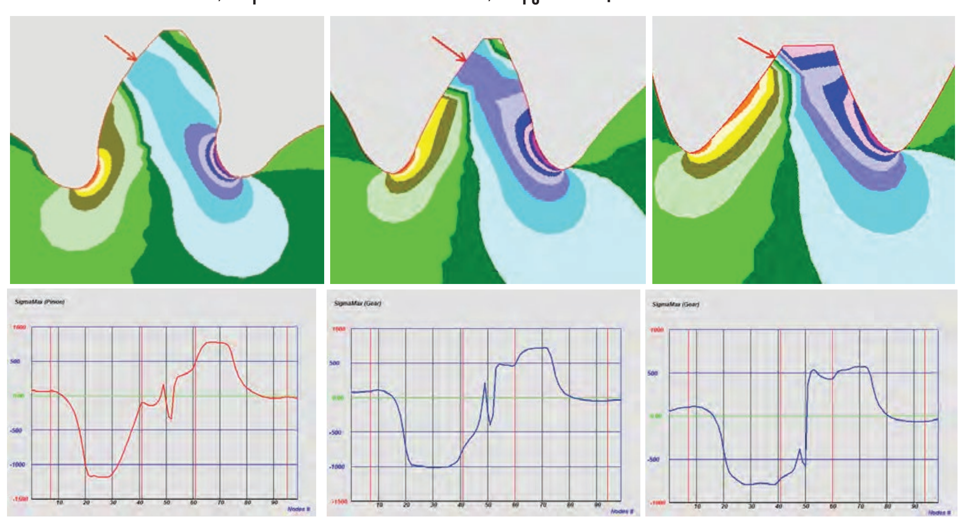
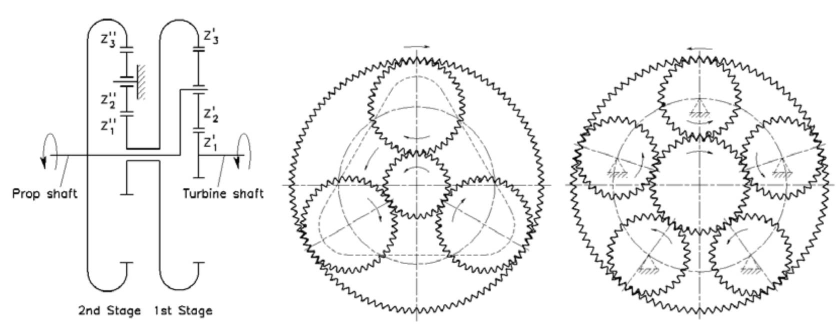


Figure 7: (a) Asymmetric epicyclic gear stage stress isograms; (b) charts of sun gear, planet gear, and ring gear respectively



a b c

Figure 8: (a) TV7-117S gearbox arrangement; (b) first and (c) second stage sections

opposite direction. After the specified number of iterations, the optimization process stops, outputting the optimized fillet profile. During the optimization process, the fillet nodes cannot be moved inside the initial fillet profile because this may cause interference with the mating gear tooth tip.

Figure 6 compares gear tooth stress distribution before and after root fillet optimization.

Asymmetric epicyclic gear stage stress isograms and charts are shown in Figure 7.

IMPLEMENTATION OF ASYMMETRIC EPICYCLIC TOOTH GEARS

The first known application of epicyclic gears with asymmetric teeth was for the TV7-117S turboprop engine gearbox. The engine and gearbox were developed by Klimov Corporation (St. Petersburg, Russia) for the commuter airplane Ilyushin Il-114.

The TV7-117S gearbox arrangement is shown in Figure 8. The first planetary-differential stage has three planet gears. The second solar-type coaxial stage has five planet gears and stationary planet carrier. This arrangement is proved to provide the highest power transmission density for the required gear ratio.

The main parameters and characteristics of this gearbox are described in Reference [3]. The first stage (without the sun gear) and the second stage carrier assembly of the TV7-117S turboprop engine gearbox are shown in Figure 9.

a b

Figure 9: Assemblies of (a) first and (b) second stages of TV7-117S turboprop engine gearbox

CONCLUSION

This article outlines the Direct Gear Design approach to asymmetric epicyclic gear stages with the singular and compound planet gears. Methods of optimization of the tooth flank asymmetry factor and root fillet profile are considered, and an example implementation of asymmetric epicyclic gears has been demonstrated.

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